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A Study on Mathematical Modeling of India's Population Growth

Dr. P C Srivastava

Associate Professor & Head, Department of Mathematics, DAV PG College Azamgarh

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ABSTRACT

The goal of this research is to use several mathematical models to create a quantitative method for forecasting population growth. Difference equation models, sometimes known as "finite population models," have been the main application of population growth curves in recent years. Compared to the integrated versions of these models, these models might offer a slightly greater range of applications and theoretical possibilities. There are concerns over the difference equation versions' applicability in practice, though, as the integrated versions that are currently available seem to fit actual growth curve data somewhat better. In this research, we thus study several integrated and difference equation models. Although its scope has expanded recently, it is the study of long-term and marginal changes in the number of individuals in one or the other, individual weight, and age structure, with a history spanning more than 220 years. In the study of population dynamics and issues in the ecological and environmental sciences, mathematical and computational methods offer strong instruments and methods. The topic has a long history and is linked to dynamical system theory and development statistics. These mathematical and computational methods are today thought to be the most effective means of teaching natural phenomena. An interest in the study of survival and interactions between live organisms and their surroundings has been sparked by these methods, which have been widely used and have offered a framework for the synthesis and analysis of such biological models.

INTRODUCTION

Population growth has emerged as one of the world's most significant and well-known problems. Population growth is essential as a logical foundation for decision-making since the size and growth of a nation's population have a direct impact on its economy, culture, education, and environment. Accurately estimating the number of the population in the future aids in planning. For their planning efforts, all governmental and collective sectors constantly need a precise estimate of the future size of different entities, such as population, resources, needs, and consumptions [1]. In order to get this information, statisticians and mathematicians first analyze the behavior of the related variables using historical data, and then they project the desired variable in the future based on the findings of the analysis. People will have to deal with a shortage of resources in many areas, including land, minerals, energy, food, healthcare, and education, as a result of the population expansion. The improvement of people's lives, social development, and economic construction will all be significantly impacted by this. Therefore, one of our current priorities and top priorities is to strictly manage population increase. For instance, state and national population growth can be used to estimate future water demands, plan for future Social Security and Medicare commitments, and assess the need for new public schools and fire site selection. The number of people with impairments, the number of condemned offenders, and the demand for housing can all be predicted using population predictions. Population expansion is the cause of all these issues.

Mathematical modeling is a broad, multidisciplinary field that models and clarifies phenomena in the biological sciences using mathematical and computational methods. Therefore, it is a method of employing mathematical language to simulate reality. A mathematical model is described as "a collection of equations based on quantitative description of a real-world phenomenon; it is created in the hope that the predicted behaviour will resemble the actual one" [2].

A lot of people use mathematical modeling, experimentation, or observation to study population growth. Differential equations, statistical models, and dynamical systems are only a few examples of the various types of mathematical models. The Verhulst Logistic Growth Model and the simple exponential growth model are the two most well-known models in population growth estimation. In a perfect world with limitless resources, population growth can be adequately represented by the straightforward exponential growth model. In contrast, limiting factors in nature cause the population growth rate to

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steadily decline as population size increases. At the same time, the rate of population expansion decreases and eventually stops. We call this Verhulst logistic expansion. [3] [4]. The carrying capacity (K), or the maximum number of members of a given population that the environment can sustain, is typically used to describe the population size at which growth stops [3]. It is anticipated that in an ecosystem that can sustain a finite population, the rate of population growth will slow down as the limiting population gets closer. The Verhulst-logistic model is a suitable model that is provided by

$$\frac{dN}{dt} = rN\left(1-\frac{N}{N_m}\right)$$

R is the intrinsic growth rate, and Nm is the maximum population that can be maintained.

Assuming that N ? N0 at t = 0, we can solve this equation for N.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{N_m}\right)$$

$$\text{Let } x = \frac{N}{N_m} \qquad \text{ie. } N = N_m x$$

$$N_m\frac{dx}{dt} = rN_m x(1-x)$$

$$\frac{dx}{dt} = rx(1-x)$$

$$\frac{dx}{x(1-x)} = rdt$$

$$\text{that is } \left[\frac{1}{x}+\frac{1}{1-x}\right]dx = rdt$$

Integrating both sides we get

$$\ln\left|\frac{x}{1-x}\right| = rt+c$$

Using the boundary conditions that we have

$$t=0, \quad x = x_0 \quad \left(=\frac{N_0}{N_m}\right)$$

$$\therefore \quad \ln\left|\frac{x}{1-x}\cdot\frac{1-x_0}{x_0}\right| = rt$$

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calculating both sides' natural logarithms

$$\frac{x}{1-x}\cdot\frac{1-x_0}{x_0}=e^{rt}$$

$$x\left(\frac{1}{x_0}-1\right)=(1-x)e^{rt}$$

$$\frac{x}{x_0}-x=(1-x)e^{rt}$$

$$0=\frac{x}{x_0}-(1-x)e^{rt}-x$$

$$0=x\left(\frac{1}{x_0}+e^{rt}-1\right)-e^{rt}$$

$$\therefore x=\frac{e^{rt}}{\left(\frac{1}{x_0}+e^{rt}-1\right)}$$

to straighten up the expression Add both sides by ert.

$$e^{-rt}x=\frac{1}{\left(\frac{1}{x_0}+e^{rt}-1\right)}$$

$$x=\frac{1}{1+\left(\frac{1}{x_0}-1\right)e^{-rt}}$$

Now putting $x=\frac{N}{N_m}$ and $x_0=\frac{N_0}{N_m}$ back into the above expression, we get

$$\frac{N}{N_m}=\frac{1}{1+\left(\frac{N_m}{N_0}-1\right)e^{-rt}}$$

$$N=\frac{N_m}{1+\left(\frac{N_m}{N_0}-1\right)e^{-rt}}$$

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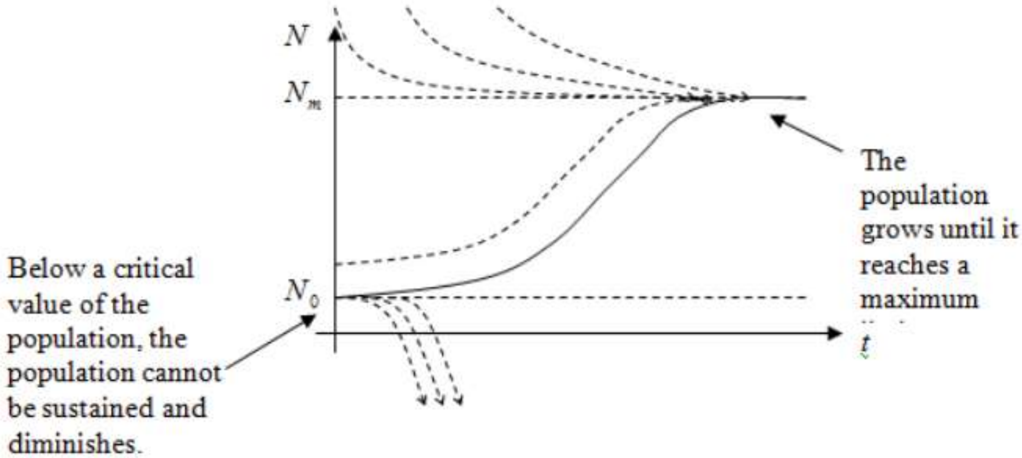
This outcome can be shown in a sketch.



The use of statistical software tools to create a statistical model for population estimation is growing in popularity and adds value [5]. Numerous websites, including [5], provided demographic projections for various nations or areas. These estimates are frequently imprecise, though, and using them for planning purposes—especially by governments—can be dangerous. Based on the information currently available, this article suggests two distinct methods for estimating a nation's or region's population. A statistical model for the population estimation rate is constructed using a statistical software program. An increasing number of people are interested in studying population growth [5]. Using a variety of mathematical models, I will review previous research on population growth in my work.

Instead of concentrating solely on the overall population level, it is helpful to take into account the pace of change when examining how the global population is changing over time. The annual population growth rate for the years 1750–2010 and projections through 2100 are placed on the global population in the following image. The greatest significant shift in population increase occurred during this time in history. The global population growth rate was consistently significantly below 1% prior to 1800. However, throughout the first half of the twentieth century, annual growth rose to as much as 2.1%, which was the greatest annual growth rate ever recorded in 1962. The growth rate has been steadily declining since peaking; estimates for 2100 indicate an annual rate of 0.1%. Accordingly, the global population will not double in the twenty-first century, even though it quadrupled in the twentieth [5].

Wali et al. [6] have noted Rwanda's population rise. Situated in Central Africa, Rwanda is a small landlocked African nation. Tanzania, Burundi, Uganda, and the Democratic Republic of the Congo are its neighbors. With 1,670 square kilometers of water and 24,668 square kilometers of land, it has a total area of 26,338 square kilometers. The majority of people live in rural areas, and the majority of the country is covered with savanna grasslands. Its population is growing at an extremely rapid rate. Ideally, nature will take over and the death rate will increase to address the issue if the population keeps growing unchecked. Regretfully, this is not the most desirable situation; in order to slow population increase, the birth rate would like to be managed. Secondary classified annual population statistics for Rwanda from 1980 to 2008 (inclusive) were gathered for his work from the National Institute of Statistics of Rwanda (NISR) and the International statistics Base (IDB). Predicted population values were calculated using MATLAB and the logistic growth mathematical model.

The graph of Rwanda's classified annual population statistics from 1980 to 2008 (inclusive) was likewise created using the Statistical Package for Social Sciences (SPSS). Thomas R. Malthus [7], an Englishman, put forth a mathematical model of population growth in 1798. In this instance, the differential equation controlling population increase is

$$\frac{d}{dt}N(t) = aN(t)$$

where a, also known as the Malthusian factor, is the multiple that establishes the growth rate and t is the time period. The Malthusian law of population expansion is a non-homogeneous linear first-order differential equation.

Using this model, Wali et al. calculated Rwanda's carrying capacity and the key variables influencing population growth. In order to make the population size proportionate to both the prior population and a new term, he altered Malthus's Model.

$$\frac{a - bN(t)}{a}$$

where a and b are referred to as the population's vital coefficients. The population's distance from its upper limit is indicated by this word. This new term will, however, become extremely small and eventually reach zero as the population value increases and approaches a/b, giving the proper feedback to restrain population expansion. The carrying capacity of the population is represented by the ratio a/b, and in the instance of Rwanda, it is estimated to be 77208025.64. The vital coefficients a and b in the Rwandan example were determined to be 0.03 and 3.885606419 × 10−10, respectively. According to this model, Rwanda's population is growing at a pace of 3% annually.

For population growth prediction, Li et al. [8] used an enhanced logistic model known as a logarithm logistic model. He contrasted many models, including the power law exponent model, the logistic growth model, and the Malthus population model (also known as the exponential growth model). Every model has benefits and drawbacks. [9]. By examining the data from the British population survey, renowned British demographer Malthus proposed the theory that the rate of population growth is constant. He then presents the exponential growth model that follows.

$$x(t) = x_0 e^{rt}$$

where x0 is the starting population and r>0 is the growth rate that hasn't changed. In the beginning, the Malthus model produces a good fit, but in the end, it produces a poor result. The exponential growth model's limit tends to infinity, which is a significant factor. Therefore, this model is only appropriate for describing the early stages of a biological population's expansion. The constant growth rate is the primary flaw in the Malthus model. We are aware that a number of factors, including population migration, living space constraints, water availability, infectious diseases, birth and mortality rates, and the impact of public health conditions, war, pollution, raw material levels, dietary patterns, and psychological stress, are closely linked to the rate of population growth. In order to rectify this mistake, Danish biomathematician Pierre-Francois Verhulst [10,11] provided an enhanced growth model in the 1840s by examining the impact of environmental factors, natural resources, and other elements. He believes that environmental factors and natural resources will prevent population increase. According to Verhulst, the growth rate is a linear function, meaning that r(x) = r - sx, (s>0, r>0), where r is the growth rate that was constant at the beginning. If the maximum population capacity is xm, then x=xm indicates that

$$r(x_m) = 0$$

Consequently, we have s=r/xm. As a result, the conventional logistic model is as follows.

$$\begin{cases} \dfrac{dx}{dt} = r\left(1-\dfrac{x}{x_m}\right)x \\ x(0) = x_0 \end{cases}.$$

The following function, known as the S curve, is its solution.

$$x(t) = \frac{x_m}{1 + \left(\dfrac{x_m}{x_0} - 1\right)e^{-rt}}.$$

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Future population estimates, the production prediction of new products in the economy, and the number of yeast cultures in a restricted amount of space are just a few of the many sectors that make extensive use of the logistic model. According to some tests, the logistic model closely resembles the true value. The authors in [12] make the assumption that the growth rate is a power law exponent, which is different from the logistic model.

$$r(x) = k(x - x_m)^{\alpha}$$

They point out that the overall number of some specific biological populations is significantly impacted by the model's power law index. The logistic model is often a useful instrument for making predictions. However, under certain circumstances, the linear function hypothesis is irrational. In actuality, as the population grows, the growth rate will decrease, and this reduction will occur more slowly. Li et al. therefore presume that

$$r(x) = r\left(1\text{-}log_{x_m} x\right)^k .$$

The growth rate curve for the conditions k=1, r=1, and xm=350 is shown in Figure 1.

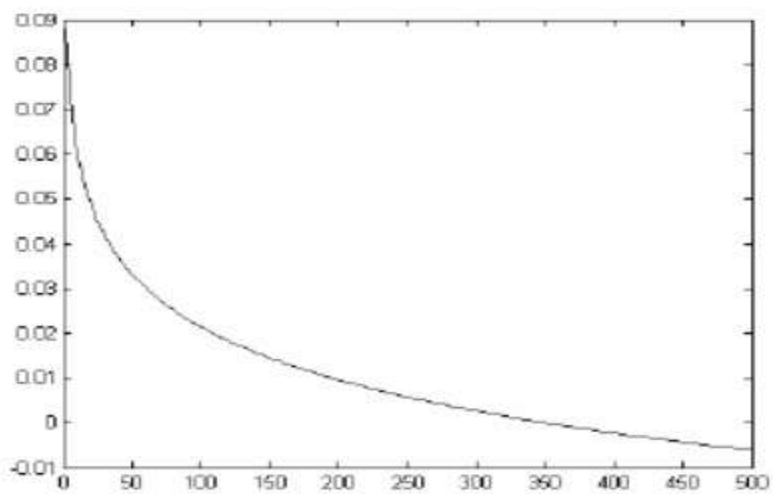

Figure 1. The curve of growth rate

Auther [8] enhanced the conventional growth model based on the fundamental population equation, resulting in the differential equation model that follows:

$$\begin{cases} \dfrac{dx}{dt} = r\left(1\text{-}log_{x_m} x\right)^k x \\ x(0) = x_0 \end{cases},$$

where r is the initial growth rate that remains constant and xm is the maximum population capacity. The enhanced logarithm logistic model is the name given to it.

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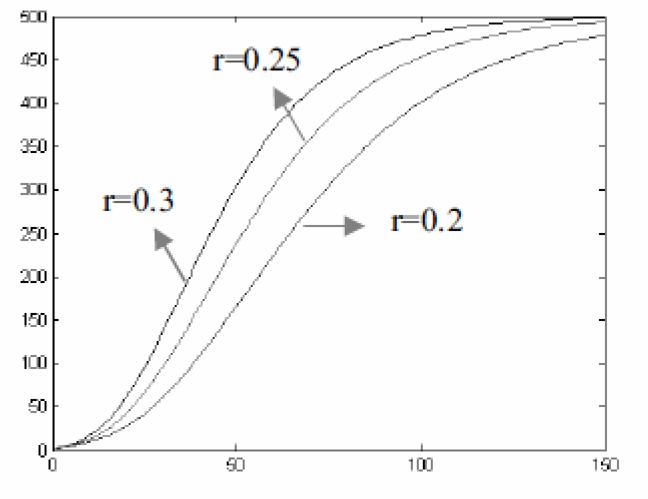

Figure 2. The logarithm Logistic curve of model (3) under different parameters

The Least Squares Method was used to estimate the model parameters based on data from the U.S. Census. The results demonstrate that compared to the traditional logistic model, the predicted value of the new models is substantially closer to the actual value. Ultimately, the best model is suggested by examining the logic of the maximum population capacity, the trend of logistic curves, and the rationality of the forecast value.

Additionally, Matintu [13] forecasted Ethiopia's population growth in 2016 using mathematical models. Next to Nigeria, Ethiopia is one of Africa's most populous nations. It borders Sudan and South Sudan to the west, Kenya to the south, Djibouti and Somalia to the east, and Eritrea to the north and northeast. Ethiopia's population growth is modeled using data from the logistic growth model and Malthus's theory. The International Data Base (IDB) provided the data that was used. We also calculate Malthus's model's optimal population growth rate using the least squares approach. The logistic growth model anticipated the carrying capacity and growth rate of per year, but the Malthus population model predicted a growth rate of per year. Both models' growth rates are in good agreement with the International Data Base's four-year growth rate projection. For both the logistic growth model and Malthus' population model, the mean absolute percentage error is calculated. This demonstrated that, out of all the models we tried, Malthus' population model appears to fit the original data the best.

Research has been done by M. Zabadi et al. [14] to create a mathematical method for forecasting Jordan's population through the year 2100. This method uses historical data from 1955 to 2016 to forecast Jordan's population using the Verhulst logistic growth equation and the simple exponential growth model. Each model's explicit solutions are precisely obtained by the application of differentiation and integration mathematical procedures. Using Minitab, a Non-Linear Regression analysis was conducted. MATLAB's curve-fitting tool (cftool) was employed. According to the results, Jordan's population was expected to reach 123.169 million by 2100, growing at a pace of 3.27% annually, according to the exponential model. According to the logistic model, Jordan's population would reach 17.346 million by 2100, growing at a pace of 4.56%. In contrast, the Verhulst growth equation projected that, with a growth rate of 5.25%, Jordan's population would reach 12.157 million by the year 2100.

CONCLUSION

an analysis of the three models' outputs showed that the logistic-based models make more sense and that the exponential model is not applicable. This research presents two updated Logarithm Logistic models after assuming that the growth rate will gradually fall as the population grows. Additionally, the models' analytical solutions are found. Lastly, the population is predicted using these models. The tests demonstrate that in some situations, our new prediction models outperform the conventional logistic model. The models can be used to describe the rise of various biological populations in addition to predicting the population.

REFERENCES

[1]. Feeney and Griffith: New Estimates and Projections of Population Growth in Pakistan, 2003, Vol.29 (3), pp.483-492.
[2]. Feller: On the Logistic Law of Growth and Its Empirical Verification. Acta Biotheoretica, 1940, Vol. 5, pp. 51-61.
[3]. Fred Brauer Carols and Castillo-Chavez: Mathematical Models in Population Biology and Epidemiology, Springer, 2001.
[4]. G. P. Samanta, C. G. Chakrabarti: On Stability and Fluctuation in Gompertzian and Logistic Growth Models. Applied Mathematics Letters, 1990, Vol. 3, pp. 119–121.
[5]. G. Pelovska And M. Iannelli: Numerical Methods for the Lotka- Mckendrick's Equation, Journal of Computational and Applied Mathematics, 2006, Vol. 197 (2), pp. 534-557.
[6]. Glen Ledder: Differential Equation: A Modeling Approach. McGraw- Hill Companies Inc. USA, 2005.
[7]. Graham Kenneth Winley: The Logistic Model of Growth, Au. J.T., 2007, Vol. 11(2), pp. 99-104.
[8]. H. Caswell and M. Fujiwara: Beyond Survival Estimation: Mark– Recapture, Matrix Population Models, And Population Dynamics, Animal Biodiversity And Conservation, 2004, Vol. 27(1), pp. 471-487.
[9]. H. LeszczyNski: Differential Functional Von Foerster Equations with Renewal, Cond. Mat. Phys., 2008, Vol. 54, pp. 361-370.
[10]. H. Smith: An Introduction to Delay Differential Equations with Applications to the Life Sciences, Springer, 2010.
[11]. H. Grafen: A Theory of Fisher's Reproductive Value, Journal of Mathematical Biology, 2006, Vol. 53(1), pp. 15-60.
[12]. Hal Caswell and Nora S´Anchez Gassen: The Sensitivity Analysis of Population Projections, Demographic Research, 2015, Vol. 33(28).
[13]. Hal Caswell: Reproductive Value, the Stable Stage Distribution, and the Sensitivity of the Population Growth Rate To Changes in Vital Rates, Demographic Research, 2010, Vol. 23(19), pp. 531-548.
[14]. Hansen: Evolution Of Stability Parameters In Single-Species Population Models: Stability Or Chaos, Theo. Popul. Biol., 1992, Vol. 42 (2), pp. 199-217.